Stochastic Scheduling and Index Policies

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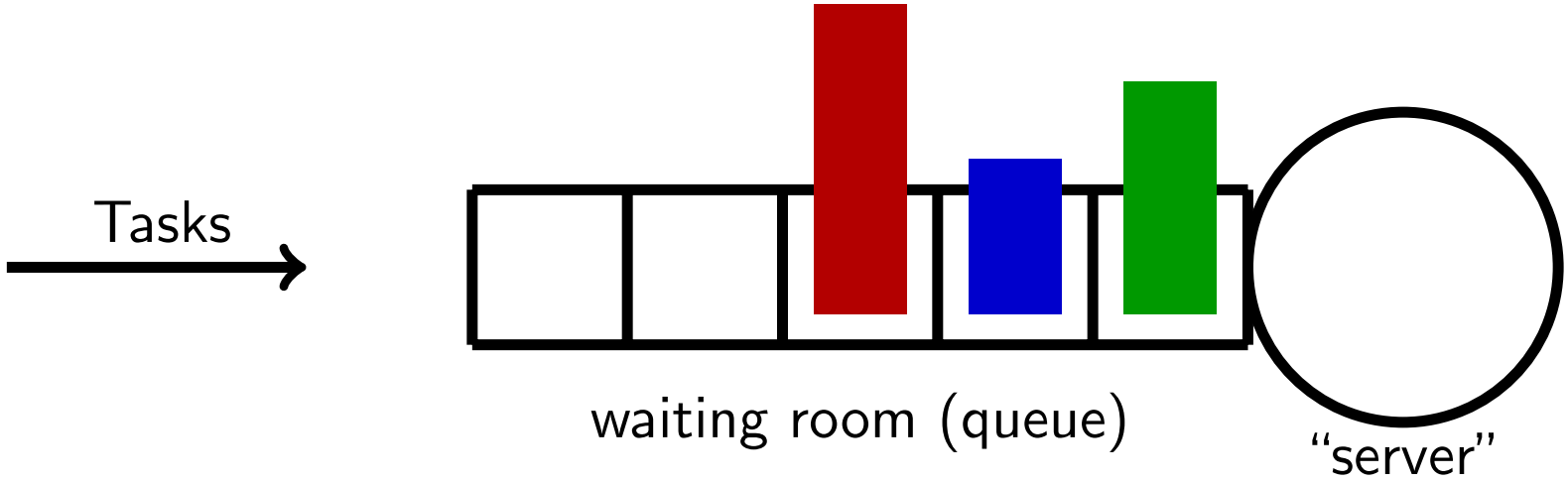
Motivation: Single-server scheduling



- We allow preemptive scheduling (preempt-resume).
- Examples: tasks allocations, electric vehicle charging, wireless scheduling, research projects,...

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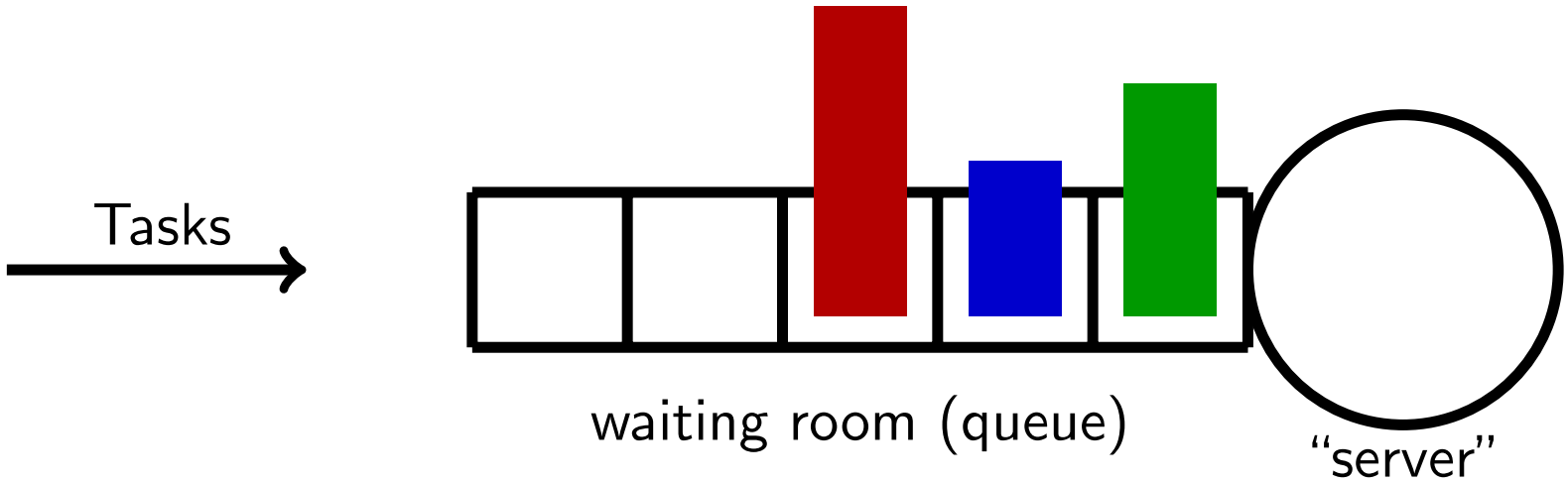

Focus of the talk

How to you schedule jobs to minimize the average response time?

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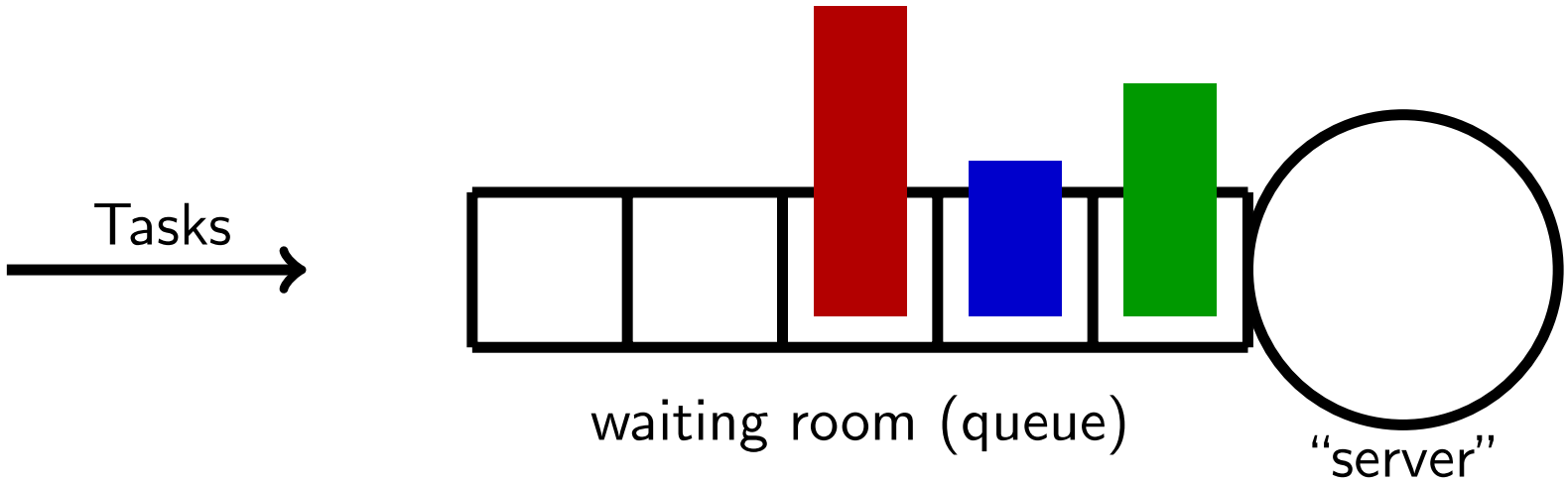

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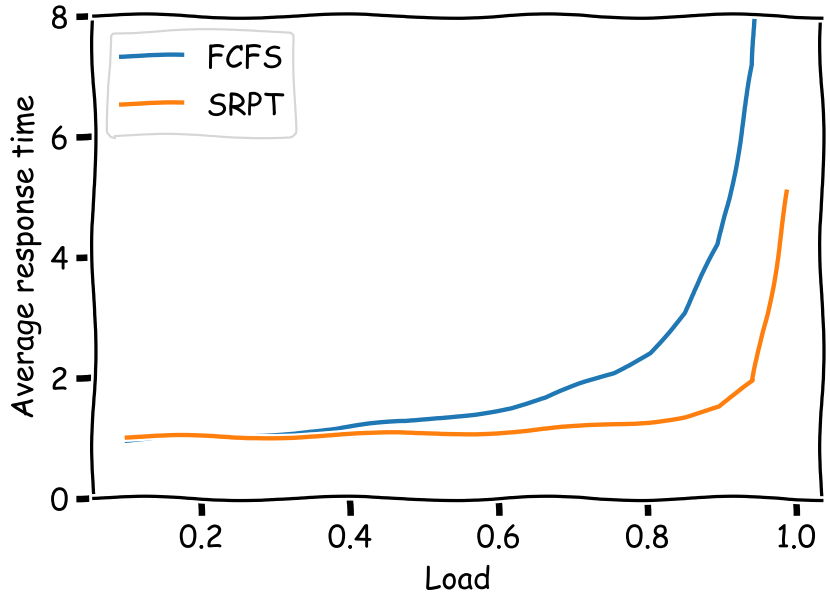
How to you schedule jobs to minimize the average response time?

If you know the job sizes: SRPT (Shortest Remaining Processing Time)

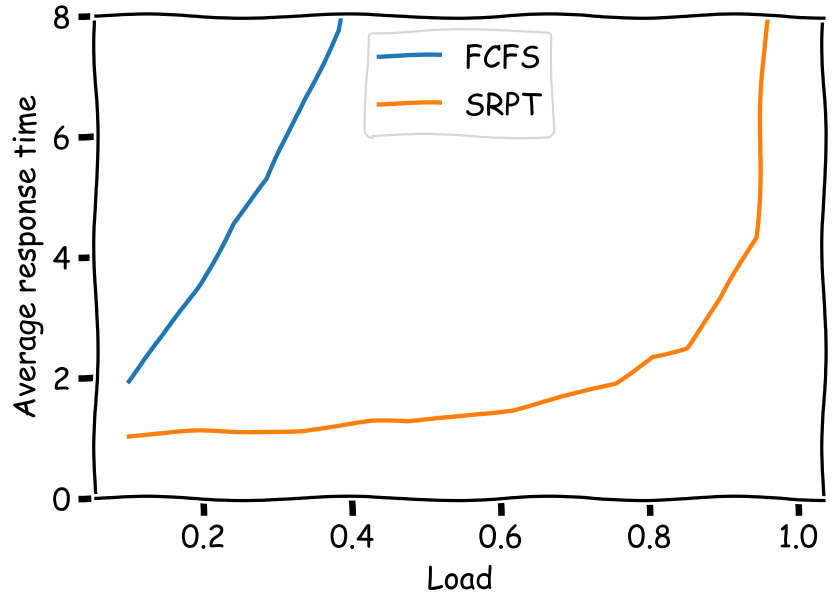
- "Strongly optimal" [Schrage, 1966]

Scheduling does matter

SRPT can lead to huge performance gain compared to FCFS



Low varying jobs
Job size $\in \{1, 10\}$
99% of small jobs.



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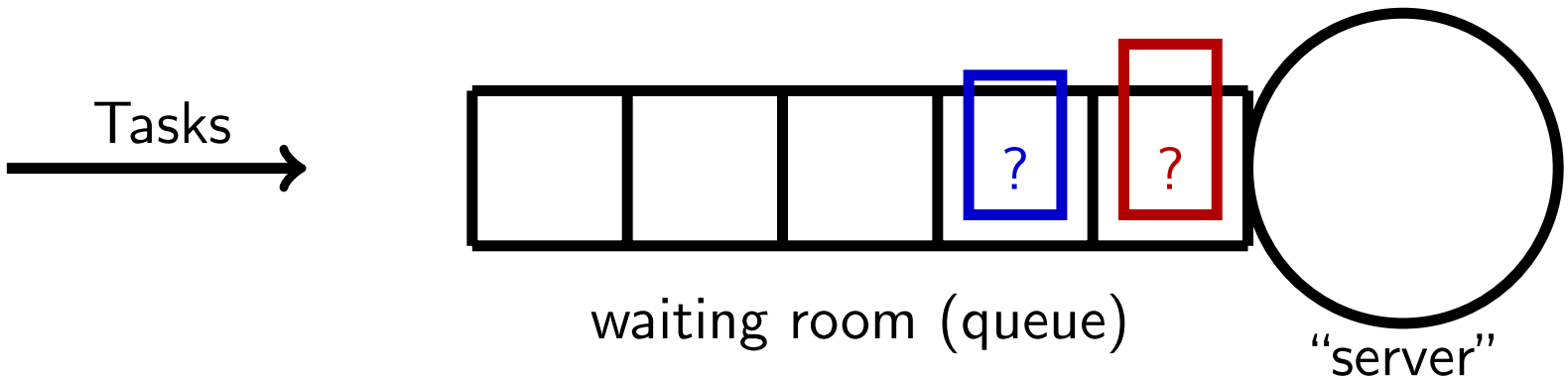
How to schedule if you do not know jobs sizes?

But you know their distribution



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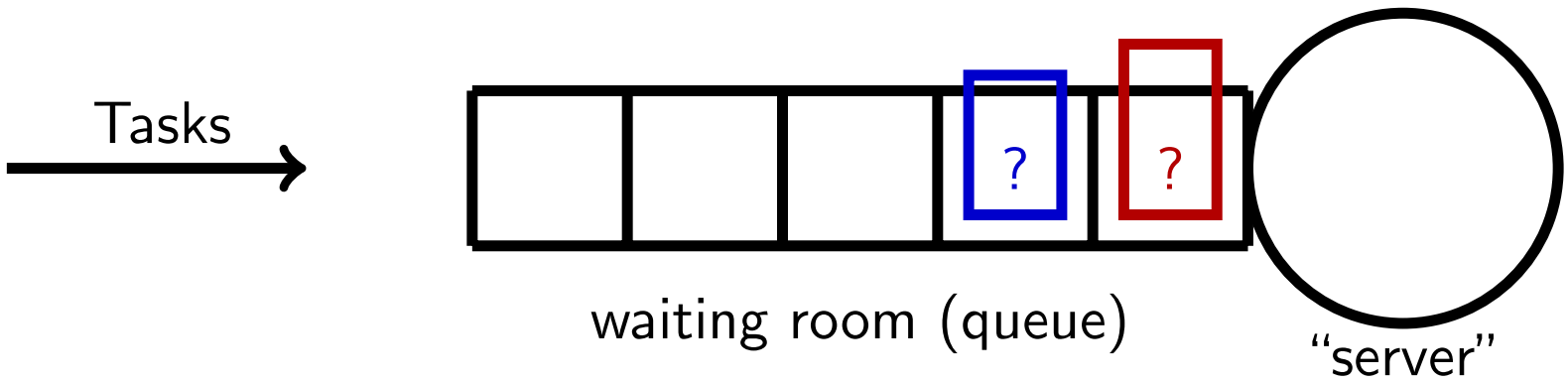
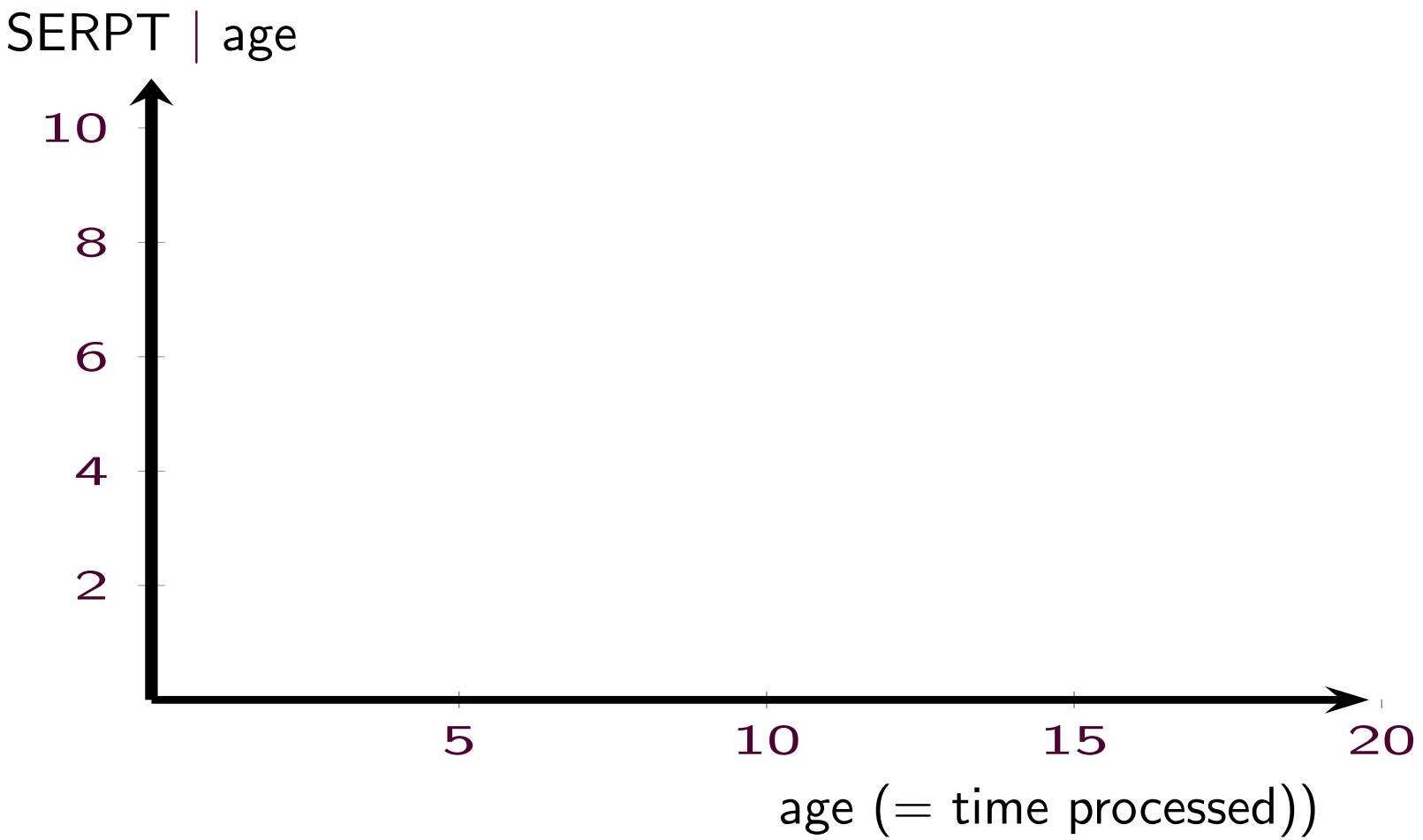


Tentative: SERPT : shortest **expected** remaining processing time.

Is it optimal?

SERPT is not optimal.

Example: two jobs with $X = 10 - \varepsilon$ and $Y = \begin{cases} 2 & \text{proba } 1/2 \\ 18 & \text{proba } 1/2 \end{cases}$



Gittins' index

If a finished job is subsidized x€ and running it costs 1€/sec.
Gittins index of a job = smallest x so that you start running it.

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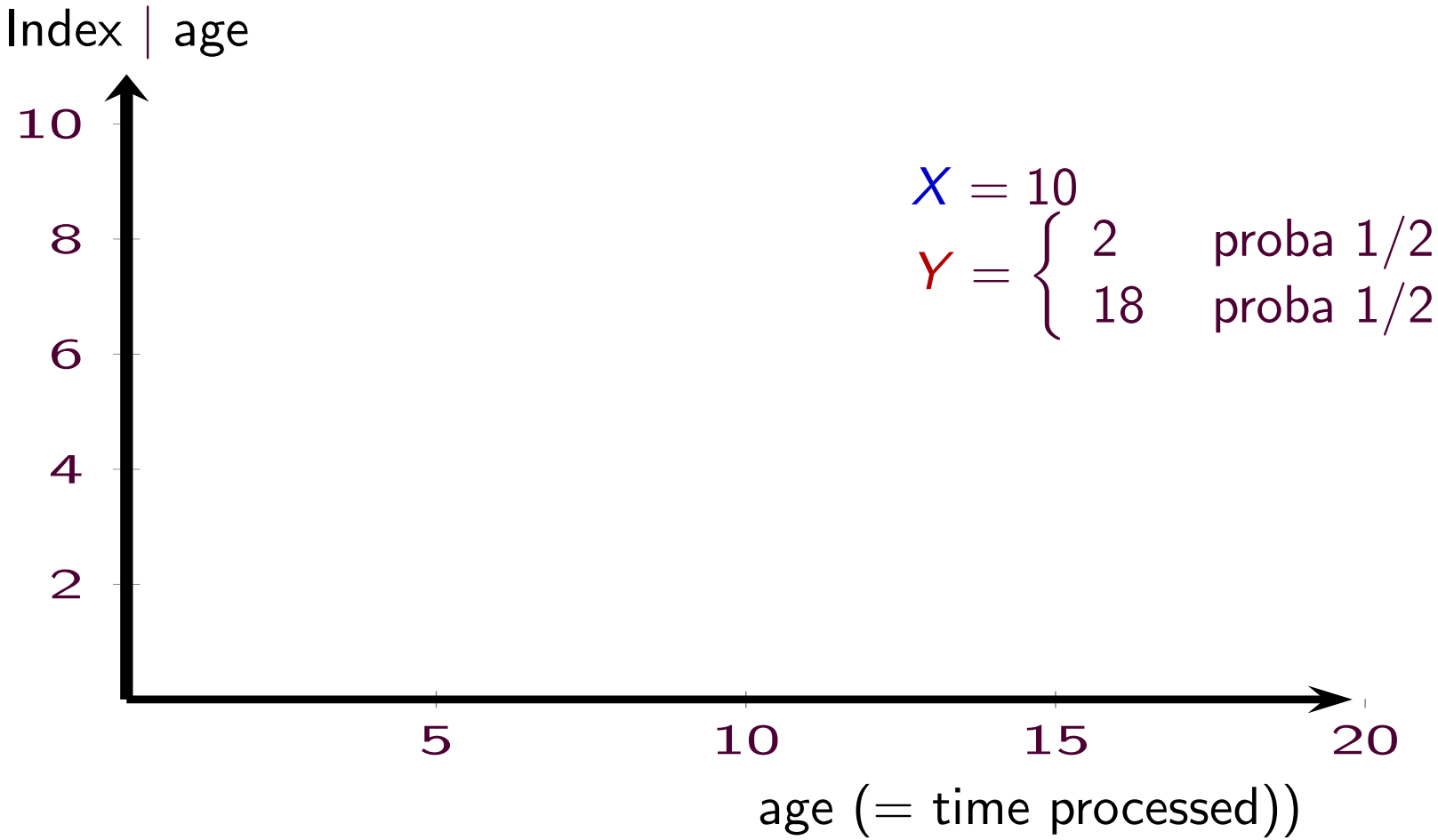

Optimality of Gittins index

Theorem. (Gittins, Glazebrook, Weber, 90s). Scheduling smallest index first is optimal.

The result is powerful but not very robust result: Gittins is not optimal as soon as:

- Multi-server, Impatient jobs,. . .

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(Recently) closed questions:

- Can we analyze the performance of this Gittins index? (SOAP: [Scully et al 2018]).
- Multi-server? (Close to optimal: [Grosof et al 2019]).
- Impatient customers? We need to change the index: [Whittle 88]. (Very close to optimal [G,G,Yan 2021])
- Unkown distribution (No regret learning.[G,G,Khun, 2021])

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"Therem" [G,G,Yan, 2021] Under quite general condition, when the number of servers n grow, Whittle index becomes asymptotically optimal exponentially fast in n.

Conclusion

Index policies are very efficient to share a single resource among tasks.

- Idea: for a given task, find the largest subsidy so that you are willing to execute this task.
- Schedule cheapest tasks in priority.
- This scales well and performs very well in practice.

Some open questions:

- Does not work "as is" when you have multiple resources : need for other relaxations.
- Does not work for "bin-packing"-like constraints.