(Refined) Mean Field Approximation for Heterogeneous Interaction Models

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Motivation

Dispatcher

Poisson Arrivals of
Rate $n\lambda$

Servers with varying processing speed

$\mu_1 \mu_2 \mu_3 \mu_{n-1} \mu_n$
Classical Mean Field Setting

Mean Field methodology:

- $M_s^{(n)}(t) = \frac{1}{n} \{ \# \text{ objects in state } s \text{ at time } t \}$
- $\lim_{n \rightarrow \infty} M_s^{(n)}(t) = \text{ODE}$

Works for:
- objects w/ homogeneous transitions
- groups of objects w/ similar statistical behavior
Need to Model Heterogeneity

Importance of Heterogeneity

• heterogeneity has a dramatic impact
e.g. for caching, epidemic modelling, load balancing
• many homogeneous models ignore heterogeneity
• in general no theoretical guarantees for accuracy

Key Question
Can we adapt and justify the mean field approximation as valid technique?
Results

\( X^{(n)}(t) \sim \text{Stochastic System} \quad X^{(n)}_{(k,s)}(t) \sim \text{indicates if item } k \text{ is in state } s \text{ as time } t \)

\[
\frac{d}{dt} \mathbb{E}[X^{(n)}_{(k,s)}(t)] = \cdots \mathbb{E}[X^{(n)}_{(k,s)}(t)X^{(n)}_{(k',s)}(t)] \cdots \\
\text{MF Assumption} \quad \approx \quad \cdots \mathbb{E}[X^{(n)}_{(k,s)}(t)] \mathbb{E}[X^{(n)}_{(k',s)}(t)] \cdots = \frac{d}{dt} \phi_{(k,s)}(t)
\]

Theorem [A., Gast, ACM SIGMETRICS 22 ]*

For the Mean Field approximation \( \phi(t) \) and refinement term \( v(t) \)

\[
\mathbb{P}(\text{item } k \text{ in state } s \text{ at time } t) = \phi_{(k,s)}(t) + O(1/n), \\
\mathbb{P}(\text{item } k \text{ in state } s \text{ at time } t) = \phi_{(k,s)}(t) + v_{(k,s)}(t) + O(1/n^2).
\]

Increasing Mean Field Accuracy as $n$ grows
Almost exact Refined MF*

* implemented using rmf_tool – a library to compute (refined) mean field approximation(s) - SIGMETRICS Perform. Eval. Rev. (Workshop)