(Refined) Mean Field Approximation for Heterogeneous Interaction Models

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POLARIS

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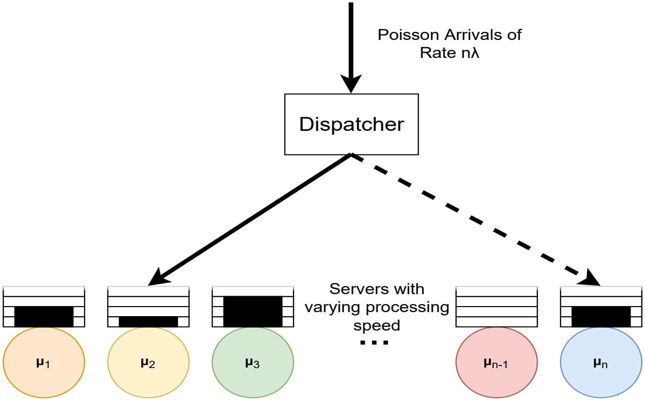
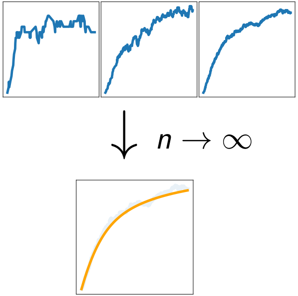
Motivation


Poisson Arrivals of
Rate nλ
Dispatcher
Servers with
varying processing
speed
μ1
μ2
μ3
μn-1
μn

Classical Mean Field Setting



Mean Field methodology:

- $M_s^{(n)}(t) = \frac{1}{n}\{\#$ objects in state s at time $t\}$
- $\lim_{n\to\infty} M_s^{(n)}(t) =$ ODE

Works for:
- objects w/ homogeneous transitions
- groups of objects w/ similar statistical behavior

Need to Model Heterogeneity

Importance of Heterogeneity

- heterogeneity has a dramatic impact
  e.g. for caching, epidemic modelling, load balancing
- many homogeneous models ignore heterogeneity
- in general **no theoretical guarantees** for accuracy

Key Question

Can we adapt and justify the mean field approximation as valid technique?

Results

$X^{(n)}(t) \sim$ Stochastic System

$X^{(n)}_{(k,s)}(t) \sim$indicates if item k is in state s as time t

$$\frac{d}{dt}\mathbb{E}[X^{(n)}_{(k,s)}(t)] = \dots \mathbb{E}[X^{(n)}_{(k,s)}(t) X^{(n)}_{(k',s)}(t)] \dots$$

$$\overset{\text{MF Assumption}}{\approx} \dots \mathbb{E}[X^{(n)}_{(k,s)}(t)]\,\mathbb{E}[X^{(n)}_{(k',s)}(t)] \dots = \frac{d}{dt}\phi_{(k,s)}(t)$$

Theorem [A., Gast, ACM SIGMETRICS 22]*

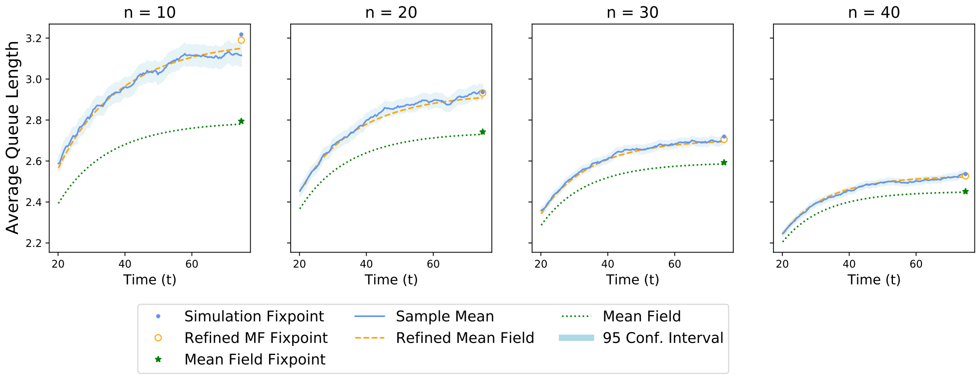
For the Mean Field approximation $\phi(t)$ and refinement term $\mathbf{v}(t)$

$$\mathbb{P}\,(\text{item } k \text{ in state } s \text{ at time } t) = \phi_{(k,s)}(t) + O(1/n),$$

$$\mathbb{P}\,(\text{item } k \text{ in state } s \text{ at time } t) = \phi_{(k,s)}(t) + v_{(k,s)}(t) + O(1/n^2).$$

* Mean Field and Refined Mean Field Approximations for Heterogeneous Systems: It Works! - Proc. ACM Meas. Anal. Comput. Syst., (Feb 2022)

Increasing Mean Field Accuracy as *n* grows
Almost exact Refined MF*



* implemented using rmf_tool – a library to compute (refined) mean field approximation(s) - SIGMETRICS Perform. Eval. Rev. (Workshop)